DIRECTORATE OF TECHNICAL EDUCATION, CHENNAI

BOARD EXAMINATIONS – OCTOBER 2017

ANSWER KEY

COURSE : BASIC ENGINEERING/SEMESTER-II **MAXIMUM MARKS : 75**

SUBJECT : APPLIED MATHEMATICS **QUESTION PAPER CODE : 251**

PART-A (Each question carries 2 marks) **(5 x 2 = 10)**

1. If a random variable 'X' has the following distribution, find the value of 'a'

X	0	1	2
$P(X)$	a	$2a$	$3a$

Solution:

Since $P(X)$ is the probability mass function.

Total probability $\sum p_i = 1$ --- 1 mark

$a + 2a + 3a = 1$

$6a = 1, a = \frac{1}{6}$ --- 1 mark

2. Write the mean of a binomial distribution.

Solution:

Mean $= np$ --- 2 marks

3. Give <u>two examples</u> of Poisson distribution.

Solution:

(i) Number of printing mistakes in a text book.

(ii) Number of defective blades in a packet of 100.

(iii) Number of air accidents in a year in a country.

(iv) Number of calls received in a mobile phone in a day.

--- 2 marks (any 2)

4. Write down the normal equation to fit a straight line $y = ax + b$

Solution:

Given $y = ax + b$

The normal equations are

$\sum y = a\sum x + nb$ --- 1 mark

$\sum xy = a\sum x^2 + b\sum x$ --- 1 mark

5. Find the slope of the tangent for the curve $y = 4x^2$ at $(1, 2)$

Solution:

Given $y = 4x^2$ at $(1,2)$

Differentiating w.r.to 'x' on both sides

$\frac{dy}{dx} = 8x$ --- 1 mark

$m = \left(\frac{dy}{dx}\right)_{(1,2)} = 8(1) = 8$

Slope of the tangent $m = 8$ --- 1 mark

6. State the conditions for the function $y = f(x)$ to be maximum at $x = a$

Solution:

Conditions for maximum value

A function $y = f(x)$ is maximum at $x = a$, if

(i) $\frac{dy}{dx} = 0 \ at \ x = a$ --- 1 mark

(ii) $\frac{d^2y}{dx^2} < 0 \ at \ x = a$ --- 1 mark

7. Find the area bounded by the curve $y = x^2$, the $X - axis$ and ordinates $x = 0$ and $x = 2$

Solution:

Required Area $= \int_a^b y \, dx$

$= \int_0^2 x^2 \, dx$ --- 1 mark

$= \left[\frac{x^3}{3}\right]_0^2 = \frac{(2)^3}{3} = \frac{8}{3}$

Required Area $= \frac{8}{3} \ sq.units.$ --- 1 mark

8. Find the complementary function of $(D^2 - 1)y = e^x$

Solution:

The auxiliary equation is $m^2 - 1 = 0$

$m^2 = 1, m = \pm 1$

$m_1 = 1, m_2 = -1$ --- 1 mark

∴ The roots are real and distinct.

Complementary Function

$y = Ae^{m_1 x} + Be^{m_2 x}$

$y = Ae^{1x} + Be^{-1x}$ --- 1 mark

PART-B (Each question carries 3 marks) **(5 x 3 = 15)**

9. If $E(X) = 12 \ and \ E(X^2) = 200$, find the value of $var(X)$.

Solution:

Given $E(X) = 12 \ and \ E(X^2) = 200$

Formula:

$Var(X) = E(X^2) - [E(X)]^2$ --- 1 mark

$= 200 - (12)^2$ --- 1 mark

$Var(X) = 56$ --- 1 mark

10. In a binomial distribution if $n = 8, \ p = \frac{1}{4}$ find its mean.

Solution:

Given $n = 8, \ p = \frac{1}{4}$

Mean $= np$ --- 1 mark

$= 8\left(\frac{1}{4}\right)$ --- 1 mark

Mean $= 2$ --- 1 mark

11. If the mean of a Poisson distribution is 4, what is the value of variance?

Solution:

Given that Mean $= \lambda = 4$

Variance $= \lambda = 4$ --- 3 marks

12. Show that $y = x^2 + x + 1$ is minimum at $x = -\frac{1}{2}$

Solution:

Given $y = x^2 + x + 1$

Differentiating w.r.to 'x' on both sides

$\frac{dy}{dx} = 2x + 1$ --- 1 mark

$\frac{d^2y}{dx^2} = 2 > 0$ --- 1 mark

When $\frac{dy}{dx} = 0$

$2x + 1 = 0,\ x = -\frac{1}{2}$ --- 1 mark

$\therefore$ When $x = -\frac{1}{2}$, the function attains the minimum.

13. Solve: $x\frac{dy}{dx} = y$

Solution:

Given $x\frac{dy}{dx} = y$

$\frac{dy}{y} = \frac{dx}{x}$ --- 1 mark

The solution is $\int \frac{dy}{y} = \int \frac{dx}{x}$ --- 1 mark

$logy = logx + logc$

$logy = log(xc)$ --- 1 mark

$y = xc,\ \frac{y}{x} = c$

14. Find the integrating factor of $\frac{dy}{dx} + y\ tan\ x = x^2$

Solution:

Given Differential Equation is of the form $\frac{dy}{dx} + Py = Q$

Where $P = tanx\ ,\ Q = x^2$ --- 1 mark

Integrating Factor $= e^{\int P dx}$ --- 1 mark

$= e^{\int \tan\ x\ dx}$

$= e^{log(sec\ x)}$

$= sec\ x$ --- 1 mark

15. Solve: $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 0$

Solution:

The auxiliary equation is $m^2 - 6m + 9 = 0$ --- 1 mark

$(m - 3)(m - 3) = 0$

$m_1 = 3,\ m_2 = 3$ --- 1 mark

$\therefore$ The roots are real and equal.

Complementary Function $y = (Ax + B)e^{mx}$

$y = (Ax + B)e^{3x}$ --- 1 mark

16. Find the particular integral of $(D^2 - 3D + 2)y = e^{-3x}$

Solution:

Particular Integral $= \frac{e^{-3x}}{(D^2-3D+2)}$ --- 1 mark

Put $D = -3$

Particular Integral $= \frac{e^{-3x}}{(-3)^2-3(-3)+2}$ --- 1 mark

Particular Integral $= \frac{e^{-3x}}{20}$ --- 1 mark

PART – C (Each division in each question carries 5 marks) (5 x 10 = 50)

17. (a) If a random variable 'X' has the following probability distribution

X	0	1	2	3	4	5	6
$P(X)$	$3a$	$2a$	$5a$	$4a$	$7a$	$8a$	11a

Find (i) 'a' and (ii) $P(1 \le X \le 4)$

Solution:

(i) Total probability $\sum p_i = 1$ --- 1 mark

$3a + 2a + 5a + 4a + 7a + 8a + 11a = 1$

$40a = 1, a = \frac{1}{40}$ --- 2 marks

(ii) $P(1 \le X \le 4) = P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4)$

$= 2a + 5a + 4a + 7a = 18a = \frac{18}{40}$

$P(1 \le X \le 4) = \frac{9}{20}$ --- 2 marks

(b) A random variable 'X' has the following probability distribution

X	-3	6	9
$P(X = x)$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$

Find the mean and the variance.

Solution:

Mean $= E(X) = x_1p_1 + x_2p_2 + \cdots + x_np_n$

$= (-3)\left(\frac{1}{6}\right) + (6)\left(\frac{1}{2}\right) + (9)\left(\frac{1}{3}\right) = -\frac{1}{2} + 3 + 3$

Mean $= E(X) = \frac{11}{2}$ --- 2 marks

$E(X^2) = {x_1}^2p_1 + {x_2}^2p_2 + \cdots + {x_n}^2p_n$

$= (-3)^2\left(\frac{1}{6}\right) + (6)^2\left(\frac{1}{2}\right) + (9)^2\left(\frac{1}{3}\right) = \frac{3}{2} + 18 + 27$

$E(X^2) = \frac{93}{2}$ --- 1 mark

$Variance = E(X^2) - [E(X)]^2$ --- 1 mark

$= \frac{93}{2} - \left(\frac{11}{2}\right)^2$

$Variance = \frac{65}{4}$ --- 1 mark

(c) Ten coins are tossed simultaneously. Find the probability of getting at least 7 heads.

Solution:

Given that Probability of getting head $p = \frac{1}{2}$
Probability of getting tail $q = \frac{1}{2}$
Number of coins tossed $n = 10$ --- 1 mark

Formula: $P(X = x) = nC_x p^x q^{n-x}$ --- 1 mark

Probability of getting at least 7 heads

$P(X \geq 7) = P(X = 7) + P(X = 8) + P(X = 9) + P(X = 10)$ --- 1 mark

$= 10C_7 \left(\frac{1}{2}\right)^7 \left(\frac{1}{2}\right)^{10-7} + 10C_8 \left(\frac{1}{2}\right)^8 \left(\frac{1}{2}\right)^{10-8} + 10C_9 \left(\frac{1}{2}\right)^9 \left(\frac{1}{2}\right)^{10-9} + 10C_{10} \left(\frac{1}{2}\right)^{10} \left(\frac{1}{2}\right)^{10-10}$

$= \left(\frac{1}{2}\right)^{10} [10C_7 + 10C_8 + 10C_9 + 10C_{10}] = \left(\frac{1}{2}\right)^{10} [120 + 45 + 10 + 1]$

$= \frac{176}{1024} = \frac{11}{64}$ --- 2 marks

18. (a) In a Poisson distribution, if $3P(X = 2) = P(X = 4)$, find (i) 'λ' and (ii) $P(X = 0)$.

Solution:

Formula: $P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$ --- 1 mark

(i) To find 'λ'

Given that $3P(X = 2) = P(X = 4)$

$\frac{3e^{-\lambda}\lambda^2}{2!} = \frac{e^{-\lambda}\lambda^4}{4!}$ --- 1 mark

$\frac{3}{2}\lambda^2 = \frac{\lambda^4}{24}$

$\frac{72}{2} = \frac{\lambda^4}{\lambda^2}$

$\lambda^2 = 36$

$\lambda = 6$ --- 1 mark

(ii) $P(X = 0) = \frac{e^{-6}(6)^0}{0!} = e^{-6}$ --- 2 marks

(b) In a normal distribution, if 'X' has mean 80 and standard deviation 10, find $P(70 \leq X \leq 100)$. Take $P(0 \leq Z \leq 1) = 0.3413$ and $P(0 \leq Z \leq 2) = 0.4772$.

Solution:

Given that Mean = μ = 80
Standard deviation= $\sigma = 10$ --- 1 mark

Formula: $z = \frac{X - \mu}{\sigma}$ --- 1 mark

To find $P(70 \leq X \leq 100)$

When $X = 70$, $z = \frac{70-80}{10} = \frac{-10}{10} = -1$

When $X = 100$, $z = \frac{100-80}{10} = \frac{20}{10} = 2$ --- 1 mark

$$P(70 \le X \le 100) = P(-1 \le Z \le 2)$$
$$= P(-1 \le Z \le 0) + P(0 \le Z \le 2)$$
$$= P(0 \le Z \le 1) + P(0 \le Z \le 2)$$
$$= 0.3413 + 0.4772$$
$$P(70 \le X \le 100) = 0.8185 \quad \text{--- 2 marks}$$

(c) Fit a straight line to the following data.

x	2	3	5	8	10
y	5	6	10	18	21

Solution:

Let $y = ax + b$ be the line of best fit. Given $n = 5$

x	y	x^2	xy
2	5	4	10
3	6	9	18
5	10	25	50
8	18	64	144
10	21	100	210
$\sum x = 28$	$\sum y = 60$	$\sum x^2 = 202$	$\sum xy = 432$

--- 2 marks

The normal equations are

$$a\sum x + nb = \sum y$$
$$a\sum x^2 + b\sum x = \sum xy$$

--- 1 mark

$28a + 5b = 60$ --- (1)
$202a + 28b = 432$ --- (2)

Solving equations (1) & (2), we get $a = 2.12, b = 0.106$ --- 1 mark

$y = 2.12x + 0.106$ be the line of best fit. --- 1 mark

19. (a) If the distance travelled by a particle in $'t'$ seconds is $s = 2t^3 + 3t^2 - 72t + 1$, find the acceleration when the velocity is zero.

Solution:

Given $s = 2t^3 + 3t^2 - 72t + 1$

Velocity $V = \frac{ds}{dt} = 6t^2 + 6t - 72$ --- 1 mark

Acceleration $a = \frac{d^2s}{dt^2} = 12t + 6$ --- 1 mark

<u>To find the acceleration when the velocity is zero</u>

$V = 0, \quad 6t^2 + 6t - 72 = 0$

Dividing by 6, $t^2 + t - 12 = 0$

$$(t + 4)(t - 3) = 0$$
$$t = -4, \ t = 3 \quad \text{--- 2 marks}$$

When $t = 3$, $a = \frac{d^2s}{dt^2} = 12(3) + 6$

$$a = 42\ unit/sec^2. \quad \text{--- 1 mark}$$

(b) Find the equation of the tangent and normal to the curve $y = 6 + x - x^2$ at (2, 4).

Solution:

Given $y = 6 + x - x^2$ at $(2, 4)$

Differentiating w.r.to 'x' on both sides

$\frac{dy}{dx} = 1 - 2x$ --- 1 mark

$m = \left(\frac{dy}{dx}\right)_{(2,4)} = (1 - 2x)_{(2,4)} = 1 - 2(2) = -3$

Slope of the tangent $m = -3$ --- 1 mark

Equation of the tangent at (2, 4) is

Formula: $y - y_1 = m(x - x_1)$

$y - 4 = -3(x - 2)$

$y - 4 = -3x + 6$

$3x + y - 10 = 0$ --- 1 ½ marks

Equation of the normal at (2, 4) is

Formula: $y - y_1 = \frac{-1}{m}(x - x_1)$

$y - 4 = \frac{-1}{-3}(x - 2)$

$3(y - 4) = 1(x - 2)$

$x - 3y + 10 = 0$ --- 1 ½ marks

(c) Find the maximum and minimum values of $y = 2x^3 + 3x^2 - 36x + 1$.

Solution:

Given $y = 2x^3 + 3x^2 - 36x + 1$

$\frac{dy}{dx} = 6x^2 + 6x - 36$

$\frac{d^2y}{dx^2} = 12x + 6$

--- 1 mark

When $\frac{dy}{dx} = 0$, $6x^2 + 6x - 36 = 0$

Dividing by 6, $x^2 + x - 6 = 0$

$(x - 2)(x + 3) = 0$

$x = 2, \quad x = -3$ --- 2 marks

Put $x = 2$ in $\frac{d^2y}{dx^2}$

$\frac{d^2y}{dx^2} = 12(2) + 6 = 30 > 0$

$x = 2$ is minimum.

Minimum value of y

$y = 2(2)^3 + 3(2)^2 - 36(2) + 1$

$y = -43$ --- 1 mark

Put $x = -3$ in $\frac{d^2y}{dx^2}$

$\frac{d^2y}{dx^2} = 12(-3) + 6 = -30 < 0$

$x = -3$ is maximum.

Maximum value of y

$y = 2(-3)^3 + 3(-3)^2 - 36(-3) + 1$

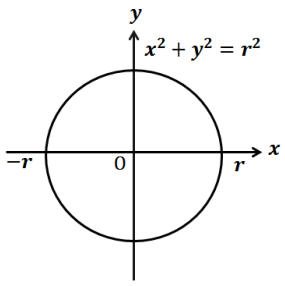
$y = 82$ --- 1 mark

20. (a) Find the volume of a sphere having radius 'r' by using integration.

Solution:

Equation of the circle is $x^2 + y^2 = r^2$ --- (1)

$y^2 = r^2 - x^2$ --- 1 mark



To find limits put $y = 0$ in Equation (1)

$x^2 = r^2$

$x = \pm r$ --- 1 mark

x varies from $-r$ to $+r$

Volume of the sphere $= \int_a^b \pi\, y^2\, dx$ --- 1 mark

$v = \pi \int_{-r}^{r} y^2\, dx$

$v = 2\pi \int_0^r y^2\, dx$ (Since function is even)

$= 2\pi \int_0^r (r^2 - x^2) dx$

$= 2\pi \left[r^2 x - \frac{x^3}{3}\right]_0^r = 2\pi \left[\frac{2r^3}{3}\right]$

$v = \frac{4}{3}\pi r^3$ cubic units. --- 2 marks

(b) Solve: $(1 + e^x) sec^2\, y\, dy - e^x\, tan\, y\, dx = 0$

Solution:

Given $(1 + e^x) sec^2\, y\, dy - e^x\, tan\, y\, dx = 0$

$(1 + e^x) sec^2\, y\, dy = e^x\, tan\, y\, dx$

$\int \frac{sec^2\, y}{\tan y} dy = \int \frac{e^x}{(1+e^x)}\, dx$ --- 2 marks

$log\ (tan\, y) = log(1 + e^x) + log\, c$ --- 1 mark

$log\ (tan\, y) = log(c(1 + e^x))$

$tan\, y = c\,(1 + e^x)$ --- 2 marks

(c) Solve: $\frac{dy}{dx} + y\, tan\, x = 4x\, cos\, x$

Solution:

The given Differential Equation is of the form $\frac{dy}{dx} + Py = Q$

Where $P = tanx$, $Q = 4x \cos x$ --- 1 mark

Integrating Factor $= e^{\int P dx}$

$= e^{\int \tan x\, dx} = e^{log(\sec x)} = sec\, x$ --- 1 mark

The general solution is

$y\, e^{\int P dx} = \int Q\, e^{\int P dx}\, dx + c$ --- 1 mark

$y\, sec\, x = \int 4x\, cos\, x\, sec\, x\, dx + c$ --- 1 mark

$y\, sec\, x = \int 4x\, dx + c$

$y\, sec\, x = \frac{4x^2}{2} + c$

$y\, sec\, x = 2x^2 + c$ --- 1 mark

21. (a) Solve: $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = 0$

Solution:

The auxiliary equation is $m^2 + 2m + 2 = 0$ --- 1 mark

$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{4-8}}{2} = \frac{-2 \pm \sqrt{-4}}{2} = \frac{-2 \pm \sqrt{4}i}{2} = \frac{-2 \pm 2i}{2} = -1 \pm i$

$\alpha = -1,\ \beta = 1$

$\therefore$ The roots are imaginary --- 2 marks

Complementary Function

$y = e^{\alpha x}(A\, cos\, \beta x + B\, sin\, \beta x)$

$y = e^{-1x}(A\, cos\, 1x + B\, sin\, 1x)$ --- 2 marks

(b) Solve: $(\boldsymbol{D^2 + 6D + 5})\boldsymbol{y} = \boldsymbol{2e^x}$

Solution:

The auxiliary equation is $m^2 + 6m + 5 = 0$

$(m + 1)(m + 5) = 0$

$m_1 = -1,\ m_2 = -5$ --- 1 mark

$\therefore$ The roots are real and distinct.

Complementary Function $y = Ae^{m_1 x} + Be^{m_2 x}$

$y = Ae^{-1x} + Be^{-5x}$ --- 1 mark

Particular Integral $= \frac{2e^x}{D^2+6D+5}$, Put $D = 1$

$= \frac{2e^x}{(1)^2+6(1)+5}$ --- 1 mark

Particular Integral $= \frac{2e^x}{12} = \frac{e^x}{6}$ --- 1 mark

The complete solution is $y = C.F + P.I$

$y = Ae^{-1x} + Be^{-5x} + \frac{e^x}{6}$ --- 1 mark

(c) Solve: $(\boldsymbol{D^2 - 4})\boldsymbol{y} = \boldsymbol{sin2x}$

Solution:

The auxiliary equation is $m^2 - 4 = 0$

$m^2 = 4,\ m = \pm 2$

$m_1 = 2,\ m_2 = -2$ --- 1 mark

$\therefore$ The roots are real and distinct.

Complementary Function $y = Ae^{m_1 x} + Be^{m_2 x}$

$y = Ae^{2x} + Be^{-2x}$ --- 1 mark

Particular Integral $= \frac{sin\,2x}{D^2-4}$, Put $D^2 = -(2)^2 = -4$

$= \frac{sin\,2x}{-4-4}$ --- 1 mark

Particular Integral $= \frac{sin\,2x}{-8}$ --- 1 mark

The complete solution is $y = C.F + P.I$

$y = Ae^{2x} + Be^{-2x} - \frac{sin\,2x}{8}$ --- 1 mark

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