Engineering Mathematics - II

Subject Code: 30022

Time : 3 hrs **Max. Marks : 75**

PART - A

I. Answer any 5 questions. **(Marks : 5 x 2 = 10)**

1. Find the equation of the circle described on the line joining points $(8, -3)$ and $(2, 4)$ as diameter.
2. Write down the condition for two circles to cut each other orthogonally.
3. Find the unit vector along the vector $2\vec{i} + 3\vec{j} - 4\vec{k}$
4. Prove that the vectors $3\vec{i} - \vec{j} + 5\vec{k}$ and $-6\vec{i} + 2\vec{j} + 4\vec{k}$ are perpendicular.
5. Define vector product of two vectors.
6. Find the value of $[\vec{i} + \vec{j},\ \vec{j} + \vec{k},\ \vec{k} + \vec{i}]$
7. Evaluate $\int (3x^2 - 5\sec^2 x + \frac{7}{x})\, dx$
8. Evaluate $\int x\, e^x\, dx$

PART - B

II. Answer any 5 questions. **(Marks : 5 x 3 = 15)**

9. Find the equation of ellipse whose major axis is x axis, centre is at origin, passes through the point $(2, 1)$ and eccentricity $\frac{1}{2}$.
10. Show that the points whose position vectors $-2\vec{i} + 3\vec{j} + 5\vec{k}$; $\vec{i} + 2\vec{j} + 3\vec{k}$; $7\vec{i} - \vec{k}$ are collinear

3. Find the projection of the vectors $2\vec{i}+3\vec{j}-\vec{k}$ on $-2\vec{i}+4\vec{j}-\vec{k}$

4. Find the area of a triangle whose adjacent sides are $\vec{i}+\vec{j}+\vec{k}$ and $\vec{i}+2\vec{j}-3\vec{k}$

5. If $\vec{a}=\vec{i}+\vec{j}+\vec{k}$, $\vec{b}=2\vec{i}+3\vec{j}$ and $\vec{c}=3\vec{i}-\vec{k}$ find $\vec{a}.(\vec{b}\times\vec{c})$

6. Evaluate $\int(\tan x+\cot x)^2\,dx$

7. Evaluate $\int\frac{dx}{9+4x^2}$

8. Evaluate $\int x^2\sin x\,dx$

PART - C

(Marks : 5 x 10 =50)

i) Answer any two subdivisions in each questions

ii) All questions carry equal marks

III. a) Show that the point (9 , 2) lies on the circle $x^2+y^2-6x-10y-11=0$. Find the other end of diameter through the point.

b) Show that the circles $x^2+y^2+2x-4y-3=0$ and $x^2+y^2-8x+6y+7=0$ touch each other.

c) Find the axis, vertex, focus, directrix, equation of latus rectum, length of latus rectum of $x^2-2x+8y+17=0$.

IV. a) Show that the points whose position vectors are $2\vec{i}+4\vec{j}+3\vec{k}$, $4\vec{i}+\vec{j}-4\vec{k}$ and $6\vec{i}+5\vec{j}-\vec{k}$ form a right angled triangle.

b) Find the angle between the vectors $\vec{i}+\vec{j}+\vec{k}$ and $3\vec{i}-\vec{j}+2\vec{k}$

c) A particle acted on by the forces $3\vec{i} - 2\vec{j} + 2\vec{k}$ and $2\vec{i} + \vec{j} - 3\vec{k}$ displaced from the point $\vec{i} + 3\vec{j} - \vec{k}$ to the point $4\vec{i} - \vec{j} + 2\vec{k}$. Find the workdone.

V. a) Find the unit vector perpendicular to $2\vec{i} - \vec{j} + \vec{k}$ and $3\vec{i} + 4\vec{j} - \vec{k}$. Also find the sine of the angle between them.

b) Find the moment of force $3\vec{i} + 4\vec{j} + 5\vec{k}$ acting through the point $\vec{i} - 2\vec{j} + 3\vec{k}$ about the point $4\vec{i} - 3\vec{j} + \vec{k}$

c) If $\vec{a} = 2\vec{i} + 3\vec{j} - \vec{k}$, $\vec{b} = \vec{i} + \vec{j} + \vec{k}$, $\vec{c} = \vec{j} + 2\vec{k}$ and $\vec{d} = 3\vec{i} - 2\vec{j}$ find $(\vec{a} \times \vec{b}).(\vec{c} \times \vec{d})$

VI. a) Evaluate i) $\int (2x-1)(5x^2 + 6x - 7)\,dx$ ii) $\int \frac{\sin^2 x}{1 - \cos x} dx$

b) Evaluate i) $\int \cos^2 2x dx$ ii) $\int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$

c) Evaluate i) $\int \frac{dx}{16 + x^2}$ ii) $\int \frac{dx}{(2x+3)^2 - 25}$

VII. a) Evaluate i) $\int x \sin x\, dx$ ii) $\int x^n \log x\, dx$

b) Evaluate i) $\int x^2 \cos 2x\, dx$ ii) $\int x^2 e^{3x} dx$

c) Evaluate i) $\int_1^2 (x - x^2) dx$ ii) $\int_0^{\pi/2} \sin^2 x\, dx$

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PART - A

I. Answer any 5 questions. (Marks : 5 x 2 = 10)

1. Find the centre and radius of the circle $(x-3)^2+(y-2)^2=36$.
2. Find the equation of parabola with focus $(2,-3)$ and directrix $y-2=0$.
3. If the position vectors of the points A and B are $\vec{i}+2\vec{j}-3\vec{k}$ and $3\vec{i}+5\vec{j}-2\vec{k}$ find $|\vec{AB}|$
4. If the vectors $2\vec{i}+\vec{j}-5\vec{k}$ and $p\vec{i}+3\vec{j}-2\vec{k}$ are perpendicular to each other find the value of p.
5. Prove that $(\vec{a}-\vec{b})\times(\vec{a}+\vec{b})=2(\vec{a}\times\vec{b})$
6. Find the value of $\vec{k}\times(\vec{j}\times\vec{i})$.
7. Evaluate $\int \log x dx$
8. Evaluate $\int_1^2 (2x+5)dx$

PART - B

II. Answer any 5 questions. (Marks : 5 x 3 = 15)

1. Find the centre and radius of the circle $x^2+y^2+2x-4y+3=0$.
2. Find the equation of ellipse whose centre is (1, 2) one of the foci is (1, 3) and eccentricity $\frac{1}{2}$.

3. If $\vec{a}+\vec{b}+\vec{c}=0$ and $|\vec{a}|=3$, $|\vec{b}|=5$, $|\vec{c}|=7$ find angle between $\vec{a}$ and $\vec{b}$

4. Find the area of the triangle two of whose sides are $3\vec{i}+2\vec{j}-\vec{k}$ and $\vec{i}-2\vec{j}+\vec{k}$

5. If $\vec{a}=\vec{i}+2\vec{j}+\vec{k}$, $\vec{b}=3\vec{i}-\vec{j}+2\vec{k}$, $\vec{c}=-\vec{i}+\vec{j}$ find $[\vec{a}.\vec{b},\vec{c}]$

6. Evaluate $\int \sec x(\sec x+\tan x)\,dx$

7. Evaluate $\int \frac{(1+\log x)^{11}}{x}dx$

8. Evaluate $\int x^n \log x\,dx$

PART - C

(Marks : 5 x 10 =50)

i) Answer any two subdivisions in each questions

ii) All questions carry equal marks

III. a) Find the equation of the circle two of whose diameters are $2x+y=4$ and $3x-y=1$ and whose radius is 9 units.

b) Show that the circles $x^2+y^2-2x+6y+6=0$ and $x^2+y^2-5x+6y+15=0$ touch each other.

c) Find the axis, vertex, focus, directrix, equation of latus rectum, length of latus rectum of $(y+2)^2=-8(x+1)$

IV. a) Show that the points whose position vectors $3\vec{i}-\vec{j}-2\vec{k}$, $5\vec{i}+\vec{j}-3\vec{k}$, $6\vec{i}-\vec{j}-\vec{k}$ form an isosceles triangle.

b) Prove that the vectors $\vec{a}=\vec{i}-2\vec{j}-\vec{k}$; $\vec{b}=\vec{i}-\vec{j}+3\vec{k}$ and $\vec{c}=-7\vec{i}-4\vec{j}+\vec{k}$ are mutually orthogonal .

c) A particle acted on by the force $4\vec{i}+3\vec{j}+2\vec{k}$ is displaced from the point $2\vec{i}+3\vec{j}+\vec{k}$ to the point $6\vec{i}+5\vec{j}+8\vec{k}$ Find the workdone by the force

V. a) Find the unit vector perpendicular to $2\vec{i}+\vec{j}+\vec{k}$ and $\vec{i}+2\vec{j}+\vec{k}$. Also find the sine of the angle between them.

b) Find the torque about the point $\vec{i}+2\vec{j}-\vec{k}$ of a force represented by $3\vec{i}+\vec{k}$ acting through the point $2\vec{i}-\vec{j}-3\vec{k}$

c) If $\vec{a}=2\vec{i}+3\vec{j}+\vec{k}$, $\vec{b}=\vec{i}-2\vec{j}+3\vec{k}$ and $\vec{c}=3\vec{i}+2\vec{j}-5\vec{k}$ verify that $\vec{a}\times(\vec{b}\times\vec{c})=(\vec{a}.\vec{c})\,\vec{b}-(\vec{a}.\vec{b})\,\vec{c}$

VI. a) Evaluate i) $\int(x^2-3x+1)\,dx$ ii) $\int\frac{dx}{1+\sin x}$

b) Evaluate i) $\int \operatorname{cosec}(4x+3)\cot(4x+3)\,dx$ ii) $\int\frac{x^2-2}{x^3-6x+4}dx$

c) Evaluate i) $\int\frac{dx}{5+4x^2}$ ii) $\int\frac{dx}{\sqrt{49-x^2}}$

VII.a) Evaluate i) $\int x^2\log x\,dx$ ii) $\int x\cos 3x\,dx$

b) Evaluate i) $\int x^2e^x\,dx$ ii) $\int x^2e^{-2x}\,dx$

c) Evaluate i) $\int_0^{\frac{\pi}{2}}\frac{\cos^2 x}{1+\sin x}dx$ ii) $\int_0^1\frac{\sin^{-1}x}{\sqrt{1-x^2}}dx$